A SYSTEM APPROACH TO THE EINSTEIN-DE SITTER MODEL OF EXPANSION OF THE UNIVERSE

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Abstract

It is shown that seeing the universe as a system is compatible not only with the standard hot big bang model for a flat universe but also with the classical exponential models (inflationary, steady-state expansion). One just has to assign the appropriate value to two parameters in agreement with the equation of state in force. An example of computation of the expansion up to now is given. It combines an inflationary stage with the radiation-dominated era followed by the matter-dominated era. The approach offers a means to conciliate the standard model with high values of the Hubble constant if such values were to be definitively confirmed.

Key-words
System - Ageing - Cosmological models - Hubble constant

"...If the world has begun with a single quantum, the notions of space and time would altogether fail to have any meaning at the beginning...Clearly the initial quantum could not conceal in itself the whole course of evolution...The whole matter of the world must have been present at the beginning, but the story it has to tell may be written step by step." (these words are taken from a seminal text published by G. Lemaître in 1931 in Nature)
1. Introduction
In the present contribution, we will show that it is possible to encounter the idea of above mentioned text of a step by step written story just by considering the universe as an 'ageing system'. Therefore, we will not take the classical approach which consists in combining General Relativity with equations of physical states at different times to deduce models describing the time evolution of the universe. What we do is to reverse the approach: we start considering the universe as a system and using Einstein's field equations, we show that the two kinds of classical models (polynomial and exponential) are found back.

We recall that, according to the standard model\(^2\), the universe is homogeneous and isotropic and is governed by Einstein's field equations\(^3\), which using a Robertson-Walker metric reduce in a fashionable way to the following two (where the cosmological constant has been set to : \(\Lambda=0\))\(^2\):

\[
\frac{8\pi G}{3c^2} \rho = \frac{k}{R^2} + \frac{\dot{R}^2}{c^2R^2}
\]

(1)

\[
\frac{8\pi G}{c^4} p = -\frac{k}{R^2} - \frac{\dot{R}^2}{c^2R^2} - \frac{2\ddot{R}}{c^2 R}
\]

(2)

where: \(R(t)\) = cosmic distance scale factor, \(k\) = spatial curvature factor, \(G\) = universal gravity constant, \(p\) = pressure, \(\rho\) = density, \(c\) = speed of light. A point above the parameter means a time derivative.

In order to build a cosmological model, these two equations have to be combined with an equation of state giving the pressure \(p=p(\rho)\).

For the sake of simplicity in the demonstration, we will concentrate on the case with \(k=0\), i.e. the Einstein-de Sitter (EdS) model for a flat universe. We then find from Eq. (1) and (2) that:

\[
\rho = -\frac{\rho_c c^2}{3} (1 - 2q)
\]

(3)

where \(q\) is the deceleration parameter \((q = -\ddot{R}/R\dot{R}^2)\).

As said, we shall not follow the classical procedure in the present contribution: we impose an equation valid for systems and show that using Einstein's field equations, it is possible to find back the most current polynomial and exponential models as particular cases in a continuum.

2. Ageing of systems
The above quoted idea of G. Lemaitre of a step-by-step written story is contained in a model of system ageing. According to this model\(^4\),\(^5\), the ageing of systems results from step-by-step adaptations to operating conditions. A system is defined as a group of interlocking parts operating together\(^6\). When subjected to operating conditions, the parts and their interconnections steadily reorganize in an adaptive process and the
resulting ageing kinetics of the system as a whole follows three steps, always the same and in the same order: (1) decrease - (2) near constancy - (3) increase of the ageing rate. Ageing is taken in the sense of an evolution or progress in time, without giving any qualitative value to these words: it is just the displacement of the system along the positive time axis in a quadri-dimensional space-time. The words ageing and evolution will thus be considered synonyms in the following. Ageing systems involve mechanical systems like diesel engines, aircrafts, but also materials subject to creep. Their ageing can be modelized by the combination of many positive and negative first order feedback loops which occur in subelements of the system when time elapses as a result of the step-by-step adaptation of the system to its operating conditions in a given environment.

The schematic view of a negative first order feedback loop is showed in Fig. 1. A very simple way to understand what happens is the following. The operating of the system in a given environment results in internal challenges on the subelements. The subelements have to adapt to the challenges in due time in order to allow further operation of the system as a whole. At each time, each subelement will modulate its available operating resources in a way to give the most appropriate response y(t) to the particular challenge K it has to meet. The corresponding mathematical expression is given by:

$$y(t) = K \cdot (1 - e^{-bt})$$

where: b : reverse of time constant for the response.

![Diagram](image)

Fig.1 : Negative first order feedback loop for a subelement.

However, during operation, the challenges are not removed: they appear again and again on a steady base with a given kinetics depending on the operating conditions. It may happen that because of the kinetics, the adaptations in the subelements are not perfect even if satisfactory for the further operation of the system. This may result in loss of information for neighbouring subelements in what their challenges are. Their responses will then become less perfect. There will be a snowball effect: imperfect responses will induce imperfect assessments of the challenges which in turn will induce
other imperfect responses. This corresponds to positive first order feedback loops as showed in Fig. 2. The corresponding mathematical expression is an increasing exponential:

$$z(t) = z_0 \cdot e^{at}$$  \hspace{1cm} (5)

![Diagram of a feedback loop](image)

**Fig.2**: Positive first order feedback loop for a subelement.

The macroscopic evolution of the system with time will be the resultant of multiple combinations of negative feedback loops (corresponding to local adaptive responses) and positive feedback loops (corresponding to local imperfect assessments of challenges) on subelements.

Now, another concept appears in the literature on the reliability of systems: that of **reliability growth**. This concept corresponds to the observed fact that, for many operating systems, the failure rate firstly decreases with time toward a minimum value. Duane called this the **learning curve**. This behaviour is also referred to as corresponding to the *infant illnesses* of the system. Reliability growth will be reflected by a decrease of “b” in Eq (4). The combination of multiple negative and positive first order feedback loops taking account of reliability growth will lead to global evolution curves of the kind shown on Fig. 3 (ref. 1).

Let us illustrate it on a simple example. Take time increments of 10 units of time (u.t.). Suppose, in order to take account of reliability growth, that the “b” corresponding to the multiple negative feedback loops decrease as follows (see Fig. 4):

$$b_i(t) = \frac{1}{t_i^{0.5}} + v$$  \hspace{1cm} (6)

for:  \hspace{0.3cm} t_{i-1} < t \leq t_i (i = 1...n)

If, for example,  \hspace{0.3cm} u = 15, v = 0.01, “a” in Eq. (5) is such that the timescale of the process is 400 u.t. (a = 0.0025) and  \hspace{0.3cm} z_0 = 1, we can write:
From $t = t_0$ to $t = t_1 \left(t_1 = t_0 + 10u \cdot t\right)$,
\[ z(t) = 1 \quad \text{and} \quad y(t) = z(t) \left(1 - e^{-b(t) \cdot t}\right) \quad (7) \]

From $t = t_{i-1}$ to $t = t_i \left(t_i = t_{i-1} + 10u \cdot t\right)$,
\[ z(t) = e^{a \cdot t_{i-1}} \quad \text{and} \quad y(t) = y_{i-1} + \left(z(t) - y_{i-1}\right) \left(1 - e^{-b(t) \cdot (t_i - t_{i-1})}\right) \quad (8) \]

The evolution of $y(t)$ and $z(t)$ with time is shown on Fig. 3 and 4 respectively.

![Graph](image)

**Fig. 3**: Global evolution curve for $b_1(t)$ and $z(t)$ as given in Fig. 4.

![Graph](image)

**Fig. 4**: Chosen values of $b_1(t)$ and $z(t)$ for the calculation of $y(t)$ (see Eq. 7 & 8).

Such an evolution is equivalent to the case of a vessel whose filling rate is regulated by a float while the volume is increasing exponentially at a rate sufficiently low to allow the filling rate to decrease in a first stage. Note that above example is just one among others possible cases of combination of negative and positive feedback loops. It was chosen for the sake of simplicity. Other combinations can be made and/or other values of $u$, $v$, $a$ and $z_0$ chosen. In order to get a global curve of the kind shown on Fig. 4, it is
sufficient that: (1) reliability growth be taken into account, and (2) $\alpha$ be not that high that it masks the effect of the negative feedback loops.

What is found in the literature on the reliability of systems is not the curve of Fig. 4 but rather its derivative, which is called the *bathtub curve* because of its shape. The *bathtub curve* usually gives the failure rate in an operating system in function of time. A typical bathtub curve is shown in Fig. 5.

![Bathtub Curve](image)

**Fig. 5: Typical "bathtub" curve.**

Looking at this curve, one notices three stages: (1) a first stage where the rate of events drastically decreases with time; (2) a second stage where a stabilization occurs around a constant or slightly increasing rate of events; (3) a final stage with an accelerating increase ending with rupture or wear out of the system. This behaviour was synthesized by following differential equation $^{4,5}$ derived from works of Duane $^7$, Cox & Lewis $^8$ and Lee $^9$:

$$\frac{E(t)}{E(i)} = \alpha + \frac{\beta}{t}$$  \hspace{1cm} (9)

where: $E(t)$: measurable parameter which reflects the ageing process
$\alpha, \beta$: constants ($0 \leq \alpha, \beta \leq 1$)

It has again to be emphasized that, in an interpretation with multiple negative and positive feedback loops in subelements, Eq (9) reflects the global evolution of the system as a whole. The feedback loops occur at a level which is several orders of magnitude (say $10^6$ or $10^{10}$ or more) smaller than the level at which the system as a whole behaves. For a class of systems, the values of $\alpha, \beta$ may slightly change in function of the internal combination of feedback loops proper to each particular system.

However, the key point is that the global evolution will be described by Eq. (9) and the evolution rate will have the shape of a *bathtub curve* (or part of it). The reverse of $\alpha$ $(1/\alpha)$ gives the order of magnitude of the timescale for the evolution process. For
instance, for a main propulsion diesel engine\textsuperscript{10}, the typical lifetime will be 20-25,000 hours ($\alpha = 10^{-8} \text{ s}^{-1}$), while for a low-alloy steel creeping at a temperature of 903K and under a stress of 60 MPa, it will only be 250-350 hours ($\alpha = 10^{-6} \text{ s}^{-1}$). Typical values of $\beta$ will range from 0.2 to 0.7 for many systems\textsuperscript{11}. In the example of Fig. 3 and 4, we find: $\beta = 0.421, \alpha = 0.00252(u/t)^{-1}$.

The solution of Eq. (9) is a combination of a power law and an exponential:

$$E(t) = K_e^{\alpha t}, t^\beta$$

(10)

Therefore, we can write alternatively:

$$\dot{E}(t) = K_e^{\alpha t}, t^\beta - 1, e^{\alpha t} + K, \alpha t^\beta, e^{\alpha t}$$

(11.1)

or:

$$\dot{E}(t) = K_e^{\alpha t}, t^\beta - 1, \left(1 + \frac{\alpha}{\beta}, t\right)$$

(11.2)

Eq. (11.2) reduces to the one proposed by Lee\textsuperscript{10} when $\alpha$ is neglected v. $\beta$. This equation has recently been used to interpret acoustic emission signals for the follow-up of creep in cellular glass\textsuperscript{12}. When $\beta = 0$, equation (11.1) reduces to the equation applied by Cox & Lewis\textsuperscript{9} to aircraft air conditioners\textsuperscript{13}, submarine main propulsion diesel engines\textsuperscript{14}, etc. When $\alpha = 0$, it reduces to the equation of Duane\textsuperscript{7} which was applied for reliability growth observations of various complex aircraft devices, loading cranes\textsuperscript{13}, army trucks\textsuperscript{15}, etc. Eq. (10) can be used to describe the creep of metals\textsuperscript{16}, thermal synchronous generating units, acoustic emission signals during load tests and several other phenomena related to ageing\textsuperscript{3}.

3. Analogy with cosmology

According to a relativistic hot big bang cosmology, the universe is closed and governed by internal forces. It contains its whole energy from the beginning. If we consider the universe to be an evolving system, the interlocking force during its evolution will be gravitation. The adaptation process will lie in the fact that the universe's expansion is partly counteracted by the gravitational forces. According to this interpretation, the resulting evolution kinetics will be given by the observed expansion. The evolution parameter is then the scale factor $R(t)$. Indeed, Robertson and Walker introduced this factor in their metrics in order to have one single parameter for describing the universe's expansion in function of time. We thus will write:

$$\frac{\dot{R}(t)}{R(t)} = \alpha + \frac{\beta}{t}$$

(12)

As $H = \frac{\dot{R}}{R}$ by definition ($H$ is the Hubble constant), one has:
\[ H(t) = \alpha + \frac{\beta}{t} \]  

(13)

In the classical polynomial models, we find that \( H(t) = \beta/t \), with, for instance, \( \beta = 1/2 \) during the radiation-dominated era (sometimes called « Weinberg model ») and \( \beta = 2/3 \) during the matter-dominated era (EdS model). In the exponential models (de Sitter model, Hoyle and Bondi's model of steady universe, inflationary models) we have: \( H(t) = \alpha = \) constant in time. Therefore, using a system approach is equivalent to saying that the Hubble constant is not either (1) constant in space but not in time (polynomial models) or (2) constant in both time and space (exponential models), but rather a combination of (1) and (2). It suggests a way to combine the polynomial and exponential models when describing the universe’s expansion. What happens then?

Starting from Eq. (13), we have, at present time:

\[ H(t_0) = H_0 = \alpha + \frac{\beta}{t_0} \]  

(14)

Here and in the following, the subscript « 0 » stands for « at any particular time \( t_0 \) in the period corresponding to a given model or at present time \( t_0 \) » (obvious in the context). According to measurements during the last decade, at present time \( 50 \leq H_0 \leq 100 \text{ km sec}^{-1} \text{ Mpc}^{-1} \) \( (1 \text{ Mpc} = 3.0856 \times 10^{19} \text{ km}) \).

From Eq (12) and (14), we find:

\[ \frac{\dot{R}}{R} = H_0 - \beta \left( \frac{1}{t_0} - \frac{1}{t} \right) \]  

(15)

Integrating Eq. (15) gives:

\[ R(t) = k \cdot \exp \left[ (H_0 - \frac{\beta}{t_0})t \right] t^\beta \]  

(16)

where \( k \) = integration constant

and:

\[ R_0 = k \cdot \exp (H_0 \cdot t_0 - \beta) \cdot t_0^\beta \]  

(17)

Finally:

\[ \frac{R}{R_0} = \left[ \exp \left( \frac{t}{t_0} - 1 \right) \right]^{\frac{H_0 \cdot t_0 - \beta}{\frac{t}{t_0}}} \cdot \left( \frac{t}{t_0} \right)^\beta \]  

(18)

Therefore, it is the actual value of \( H_0 \cdot t_0 \) versus \( \beta \) which determines if \( \alpha \neq 0 \). When \( H_0 \cdot t_0 > \beta \), Eq. (18) gives a deceleration of \( R(t) \) followed by an acceleration due to the exponential factor, which is typical for the evolution of systems.

From the above results, it is easy to deduce the other cosmological parameters:
\[ q = \beta \frac{t_0^2}{t^2 \cdot (H_0 \cdot t_0 - \beta + \beta \cdot \frac{t_0}{t})^2} - 1 \]  

(19)

and:

\[ q_0 = \frac{\beta}{H_0^2 \cdot t_0^2} - 1 \]  

(20)

Using Eq. (1) with \( k = 0 \), we find:

\[ \rho = \frac{3}{8\pi G} \cdot \frac{(H_0 \cdot t_0 - \beta + \beta \cdot \frac{t_0}{t})^2}{t_0^2} \]  

(21)

and:

\[ \rho_0 = \frac{3}{8\pi G} \cdot H_0^2 \]  

(22)

and, using Eq. (2) with \( k = 0 \):

\[ p = \frac{c^2}{8\pi G} \cdot \frac{1}{t_0^2} \left[ 2 \beta - \frac{3}{2} \cdot \frac{t^2}{t_0^2} \cdot (H_0 \cdot t_0 - \beta + \beta \cdot \frac{t_0}{t})^2 \right] \]  

(23)

and:

\[ p_0 = \frac{c^2}{8\pi G} \cdot \frac{1}{t_0^2} \left( 2 \beta - 3 \cdot H_0^2 \cdot t_0^2 \right) \]  

(24)

It is worth noting that the parameters \( R/R_0, q/q_0, \rho/\rho_0, p/p_0 \) only depend on \( t/t_0, H_0 \cdot t_0 \) and \( \beta \). Their evolution can therefore easily be drawn in function of \( t/t_0 \).

As soon as an equation of state \( p = p(\rho) \) is defined, we can deduce \( q_0 \) from Eq. (3) and, using Eq. (20), the value of \( \beta \) for any known \( H_0 \cdot t_0 \).

<table>
<thead>
<tr>
<th>No</th>
<th>( \chi )</th>
<th>Equation of state</th>
<th>( q_0 )</th>
<th>( \frac{\beta}{(H_0 \cdot t_0)^2} )</th>
<th>( \alpha )</th>
<th>Model</th>
</tr>
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<tbody>
<tr>
<td>1.</td>
<td>2</td>
<td>( p = \rho t^2 )</td>
<td>2</td>
<td>3</td>
<td>( H_0 (1 - 3 \cdot H_0 \cdot t_0) )</td>
<td>Zel'dovich</td>
</tr>
<tr>
<td>2.</td>
<td>5/3</td>
<td>( p = \frac{2}{3} \rho t^2 )</td>
<td>3/2</td>
<td>5/2</td>
<td>( H_0 (1 - \frac{5}{2} \cdot H_0 \cdot t_0) )</td>
<td>‘Weinberg’</td>
</tr>
<tr>
<td>3.</td>
<td>4/3</td>
<td>( p = \frac{1}{3} \rho t^2 )</td>
<td>1</td>
<td>2</td>
<td>( H_0 (1 - 2 \cdot H_0 \cdot t_0) )</td>
<td>EdS</td>
</tr>
<tr>
<td>4.</td>
<td>1</td>
<td>( p = 0 )</td>
<td>1/2</td>
<td>3/2</td>
<td>( H_0 (1 - \frac{3}{2} \cdot H_0 \cdot t_0) )</td>
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<tr>
<td>5.</td>
<td>2/3</td>
<td>( p = \frac{1}{3} \rho t^2 )</td>
<td>0</td>
<td>1</td>
<td>( H_0 (1 - H_0 \cdot t_0) )</td>
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<tr>
<td>6.</td>
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<td>( p = \frac{2}{3} \rho t^2 )</td>
<td>-1/2</td>
<td>1/2</td>
<td>( H_0 (1 - \frac{1}{2} \cdot H_0 \cdot t_0) )</td>
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</tr>
<tr>
<td>7.</td>
<td>0</td>
<td>( p = -\rho t^2 )</td>
<td>-1</td>
<td>0</td>
<td>( H_0 )</td>
<td>Hoyle/ GAS†</td>
</tr>
</tbody>
</table>

† Guth, Albrecht & Steinhardt
The universe is usually modelled as a gas of particles in a first time, then a gas of galaxies: for such models, the equations of state are of the type \( p = (\gamma - 1) \rho \) where \( \gamma \) is an adiabatic index comprised between 0 and 2(ref.\textsuperscript{17}). The table shows seven of such equations of state and the corresponding values for \( \gamma \) and \( \rho_0 \). They correspond to a regular decrease of \( \gamma \) from 2 to 0 by decrements of 1/3. The values of \( \beta/(H_0, t_0) \)\textsuperscript{2} as deduced from Eq. (20) of the present model are given in the fifth column. The corresponding expressions of \( \alpha \) according to Eq. (14) can be seen in column 6. This parameter is directly proportional to the Hubble constant and depends on the value of \( H_0, t_0 \). Save the last case, \( \alpha = 0 \) when \( \beta = H_0, t_0 \). Otherwise, \( \alpha \neq 0 \). Some known models relevant to the selected cases are quoted in the last column. Case n°1 corresponds to a Zeldovich kind of model for stiff matter. In case n°3, if \( \beta = H_0, t_0 = 1/2 \), one finds back the « Weinberg model » for the radiation-dominated era. In case n°4, if \( \beta = H_0, t_0 = 2/3 \), one has the already quoted EdS model for the matter-dominated era. All three models are particular cases with \( \alpha = 0 \). For case n°7, it is the inverse: \( \beta = 0 \) and \( \alpha = H_0 \neq 0 \), which corresponds to a pure exponential expansion. As mentioned, such an expansion is compatible with both Hoyle's model of a steady-state universe and the inflationary model of Guth\textsuperscript{18}, Albrecht & Steinhardt\textsuperscript{19} (GAS model). Therefore, because the system approach allows to have \( \alpha \neq 0 \), the GAS Hoyle models can harmoniously become integrated in a series of models as the particular case where \( \beta = 0 \).

Now, using \( \alpha \) and \( \beta \), it is possible to describe the expansion of the universe in any relevant sequence of physical states. Let us for instance assume the following scenario (inspired by ref.\textsuperscript{20}):

- the model is valid from Planck's time \( t_p \) \( (5.38 \times 10^{-44} \text{ s}) \) after the big bang
- between \( t_p \) and \( t_1 = 5.38 \times 10^{-34} \text{ s} \), the expansion follows the Weinberg model (radiative period)
- from \( t_1 \) to \( t_2 = 5.38 \times 10^{-31} \text{ s} \), we have an inflationary step with the scale factor increasing \( 10^{50} \) times
- afterwards, the expansion is again of the radiative type up to a time \( t_3 = 10^6 \text{ years} \)
- from \( t_3 \), the EdS model applies till the present time \( t_0 \) (supposed to be \( 13.10^9 \text{ years} \) after the big bang; this value is close to the minimum measured for the oldest stars in our galaxy\textsuperscript{21}) and in the future (matter-dominated era).

For the computation, we suppose the present value of the scale factor to be equal to 13 billions years-light.

If \( \alpha = 0 \), one finds easily, starting from Eq.(5), that at present time \( t_0 \):

\[
R_0 = R_p \left( \frac{t_1}{t_p} \right)^{1/2} \exp(H_{0,2}(t_2 - t_1)) \left( \frac{t_3}{t_2} \right)^{1/2} \left( \frac{t_0}{t_3} \right)^{2/3} \tag{25}
\]

with intermediate equations.
• from $t_0$ to $t_1$ ($\alpha = 0, H_0, t_0 = \beta = 1/2$): $R = R_P \left( \frac{t}{t_p} \right)^{1/2}$ with $R_P$ the value of the scale factor at Planck's time.

• from $t_1$ to $t_2$ ($\alpha = H_{0,2}, \beta = 0$): $R = R_P \left( \frac{t_1}{t_p} \right)^{1/2} \exp(H_{0,2}, (t-t_1))$ with $H_{0,2}$ the value of the Hubble constant during the inflationary stage.

• from $t_2$ to $t_3$ ($\alpha = 0, H_0, t_0 = \beta = 1/2$): $R = R_P \left( \frac{t_1}{t_p} \right)^{1/2} \exp(H_{0,2}, (t-t_1)) \left( \frac{t}{t_2} \right)^{1/2}$

• from $t_3$ to $t_0$ ($\alpha = 0, H_0, t_0 = \beta = 2/3$):

$$R = R_P \left( \frac{t_1}{t_p} \right)^{1/2} \exp(H_{0,2}, (t-t_1)) \left( \frac{t_3}{t_2} \right)^{1/2} \left( \frac{t}{t_3} \right)^{2/3}$$  (26)

In order to have $R_0 = 13$ billions years-light, $R_P$ must be equal to $1.79 \times 10^{-19} t_p c$ (where "c" is the light speed in vacuum). If $R_P$ is smaller, the present value of the scale factor will be smaller. In order to have the scale factor to increase $10^{50}$ times during the inflationary period, $H_{0,2}$ must be equal to $6.61 \times 10^4 \text{ km/s Mpc}$. These values of $R_P$ and $H_{0,2}$ are just quoted to fix the ideas within our example.

If $\alpha \neq 0$ from $t_2$ to $t_3$ and/or from $t_3$ to $t_0$, one has (for simplicity, we take $\alpha_1 = \alpha_2$):

• from $t_2$ to $t_3$ ($\alpha = \alpha_1, H_0, t_0 = \beta = 1/2$):

$$R = R_P \left( \frac{t_1}{t_p} \right)^{1/2} \exp(H_{0,2}, (t-t_1)) \exp(\alpha_1, (t-t_2)) \left( \frac{t}{t_2} \right)^{1/2}$$

• from $t_3$ to $t_0$ ($\alpha = \alpha_2, H_0, t_0 = \beta = 2/3$):

$$R = R_P \left( \frac{t_1}{t_p} \right)^{1/2} \exp(H_{0,2}, (t-t_1)) \exp(\alpha_1, (t-t_2)) \exp(\alpha_2, (t-t_3)) \left( \frac{t_3}{t_2} \right)^{1/2} \left( \frac{t}{t_3} \right)^{2/3}$$

and Eq. (25) then becomes:

$$R_0 = R_P \left( \frac{t_1}{t_p} \right)^{1/2} \exp(H_{0,2}, (t-t_1)) \exp(\alpha_1, (t-t_2)) \exp(\alpha_2, (t_0-t_3)) \left( \frac{t_3}{t_2} \right)^{1/2} \left( \frac{t_0}{t_3} \right)^{2/3}$$  (28)

Suppose that $\alpha_1 = \alpha_2 = 3.17110^{-19} \text{ s}^{-1} (\approx 9.8 \text{ km/s Mpc})$ which corresponds to a timescale of 100 billions years for the expansion. Fig. 6 and Fig. 7 (in logarithmic scale) show the resulting variation of the scale factor with time for Eq. (26) and (27).
Fig. 6: Evolution of the scale factor of the universe following Eq. (27)
when: - curve (1): $\alpha_1 = \alpha_2 = 0$
      - curve (2): $\alpha_1 = \alpha_2 = -9.8 \text{ km/s.Mpc}$

Fig. 7: Evolution of the scale factor of the universe following Eq. (27)
(logarithmic scale)
when: - curve (1): $\alpha_1 = \alpha_2 = 0$
      - curve (2): $\alpha_1 = \alpha_2 = -9.8 \text{ km/s.Mpc}$

Curve (1) is obtained when $\alpha_1 = \alpha_2 = 0$, curve (2) when $\alpha_1 = \alpha_2 = $ the above quoted value. We observe that the EdS model is prevailing in the shape of the curves until now.
However, the third exponential in Eq. (27) will become prevailing once in the future and curve (2) will diverge from curve (1) as seen in the upper right-hand corner of Fig. 7. Let us note that a prevailing EdS model is in good agreement with recent measurements of $q_0 \approx 0.7$ estimated from the angular-size/redshift relation for compact radio sources.

**Hubble constant**

$$H(t)$$

(km/s/Mpc)

<table>
<thead>
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<th>12</th>
<th>14</th>
<th>16</th>
<th>18</th>
<th>20</th>
<th>22</th>
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<tr>
<td>(2)</td>
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</tbody>
</table>

**Fig. 8: Evolution of the Hubble constant with time**

- curve (1): $\alpha_1 = \alpha_2 = 0$
- curve (2): $\alpha_1 = \alpha_2 = -9.8$ km/s/Mpc

In Fig. 8, we check the influence of $\alpha_1, \alpha_2 \neq 0$ on the Hubble constant. As expected for the scenario of Eq. (26), we will find that $H_0 = 50$ km/s/Mpc for $t_0 = 13$ billions years (EdS model) and that higher values of $H_0$ will imply too small values of $t_0$ (e.g., $t_0 = 11.11$ years for $H_0 = 60$ km/s/Mpc) compared to the measured age of the oldest stars. However, when a scenario of the type given by Eq. (27) is taken, higher values of $H_0$ can be made compatible with the measured age of the oldest stars: but we then have $\alpha_1, \alpha_2 \neq 0$. For instance, the above values of $\alpha_1$ and $\alpha_2$

($\alpha_1 = \alpha_2 = 3.171 \times 10^{-19}$ s$^{-1}$) give $H_0 = 60$ km/s/Mpc for $t_0 = 13$ billions years and $H_0 = 50$ km/s/Mpc for $t_0 = 16$ billions years. A value of $H_0$ as high as 83 km/s/Mpc would mean that $\alpha_1, \alpha_2$ are much higher (about $\alpha_1 = \alpha_2 = 32.6$ km/s/Mpc) and the timescale of the evolution lower than mentioned above ($t_i = 30.1109$ years).

**4. Case with a non-zero cosmological constant**

In the previous analysis, we have supposed the cosmological constant to be zero. Now, the system approach can also be used with a cosmological constant different from zero. The Einstein field equations then become:
\[
\frac{8\pi G}{c^4} p + \frac{\rho c^2}{3} = \frac{2H^2}{c^2} q + \frac{2\Lambda}{3} \tag{29}
\]
\[
\frac{8\pi G}{c^4} (p + \rho c^2) = \frac{2H^2}{c^2} (1 + q) \tag{30}
\]

Combining Eq. (29) and (30) gives:

\[
\frac{8\pi G}{c^4} \left( p + \frac{\rho c^2}{3} \right) = \frac{8\pi G}{c^4} \left( p + \frac{\rho c^2}{3} \right) q + \frac{2\Lambda}{3} \tag{31}
\]

If we define:

\[
\Phi(\Lambda, \rho) = \frac{\Lambda c^2}{8\pi G \rho} \tag{32}
\]

we obtain \( \Phi = f(q) \) for the different models mentioned in the first part:

for \( p = \rho c^2 \) (Zel'dovich):

\[
\Phi = \frac{2 - q}{1 + q} \tag{33}
\]

for \( p = \frac{1}{3} \rho c^2 \) (\textit{Weinberg}):

\[
\Phi = \frac{1 - q}{1 + q} \tag{34}
\]

for \( p = 0 \) (Edd):

\[
\Phi = \frac{1 - 2q}{2(1 + q)} \tag{35}
\]

for \( p = -\rho c^2 \) (GAS-Hoyle):

\[
\Phi = -1 \tag{36}
\]

Combining Eq. (14) and (20) gives:

\[
\alpha t_0 = \sqrt{\frac{\beta}{q_0 + 1}} - \beta \tag{37}
\]

We see that \( \Phi_0 \) only depends on \( q_0 \) and \( \alpha t_0 \) on \( q_0 \) and \( \beta \). Therefore, we can draw the relationship between \( \Phi_0 \) and \( \alpha t_0 \) for the four models. For the \textit{Einstein-de Sitter model} \( (p = 0) \), for instance, we will get Fig. 9.

One finds back that \( \alpha = 0 \) for \( \beta = 2/3 \) and \( \Lambda = 0 \): this corresponds to the intersection point of the axes. However, when \( \beta \neq 2/3 \) and/or \( \Lambda \neq 0 \), we shall have \( \alpha \neq 0 \) most of the time. Similarly, fixing \( \beta \) at 2/3, we get Fig. 10 giving \( \Phi(\alpha, t_0) \) for different models. Again, we observe that, except for the particular cases where \( \Phi = 1, 1/3, 0, -1 \), we have \( \alpha \neq 0 \).
Fig. 9: Relationship between $\Phi_0 (\Lambda, \rho_0)$ and $\alpha t_0$
for different values of $\beta$

\[ \Phi_0 = \Lambda \cdot c^2 / 8\pi \rho_0 G \]
5. Conclusions

1) It is shown that a parallel can be drawn between the existing models of expansion of the universe and the evolution of systems. In particular, polynomial hot big bang models (in $t^{1.2}$ for the radiative era and $t^{2.3}$ for the matter-dominated era) and exponential models (inflationary models, steady-state universe) fit well in with an equation of evolution of systems of the type $E(t) = k_e^{-\alpha}t^\beta$ ($0 \leq \alpha, \beta \leq 1$). The polynomial part prevails when $t << 1/\alpha$ or $a$ is very close to zero while evidence of the exponential part only becomes detectable from $t \approx 1/\alpha$. It is shown that $\alpha \neq 0$ in most cases when the cosmological constant $\Lambda \neq 0$ and, for $\Lambda = 0$, when $\beta$ is (even slightly) different from its value in the classical models.

2) Under the interpretation of the expanding universe as an evolving system, we find that the Hubble constant $H = \alpha + \beta/\tau$. The existence of a part $\alpha$ also constant in time in the Hubble constant cannot be tested experimentally at present time (due to the measurement scatter) but it offers a means to conciliate two series of possibly contradictory (according to the standard model) measurement results, namely that the universe could be older than 12 Gyrs and that the Hubble constant could be higher than $H_0 = 50\text{ km s}^{-1}\text{ Mpc}^{-1}$; however, the contradiction must firstly be confirmed by definitive measurements.

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References:

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